Problem Set 3 **Set Theory & Topology** **Due 2/23/18**

You are expected to do the following problems to a high standard (i.e., at least well enough to be published in a textbook) for full credit. Four of these problems will be selected (by Jon) for grading, with each worth 5 points.

1. [Baker 3.2.7] Let $(X, \mathscr{T}), (Y, \mathscr{S})$, and $(Z, \mathscr{F})$ be topological spaces. If the function $f : (X, \mathscr{T}) \to (Y, \mathscr{S})$ and $g : (Y, \mathscr{S}) \to (Z, \mathscr{F})$ are continuous, then $g \circ f : (X, \mathscr{T}) \to (Z, \mathscr{F})$ is continuous.

2. [Baker 3.2.15] If $A \subseteq \mathbb{R}$, then a function $f : (A, \mathscr{U}_A) \to (\mathbb{R}, \mathscr{U})$ is continuous iff for each $x_0 \in A$ and each $\epsilon > 0$, there exists $\delta > 0$ such that $|x - x_0| < \delta \Rightarrow |f(x) - f(x_0)| < \epsilon$, for all $x \in A$.

3. [Baker 4.1.1] Let X and Y be sets with U and A subsets of X and V and B subsets of Y. Prove that $(A \times B) \cap (U \times V) = (A \cap U) \times (B \cap V)$.

4. [Baker 4.1.1.5] Let X and Y be sets with U and A subsets of X and V and B subsets of Y. Prove that $(A \times B) \cup (U \times V) = (A \cup U) \times (B \cup V)$ or explain why not.

5. [Baker 4.1.11] Let $(X, \mathscr{T})$ and $(Y, \mathscr{S})$ be topological spaces with $A \subseteq X$ and $B \subseteq Y$. Prove or disprove the following: $\text{Ext}(A \times B) = \text{Ext}(A) \times \text{Ext}(B)$.

6. [Baker 4.2.8] Let $(X_1, \mathscr{T}_1), (X_2, \mathscr{T}_2), ..., (X_n, \mathscr{T}_n)$ be topological spaces. If $\mathscr{B}_i$ is a base for $\mathscr{T}_i$ for each $i \in \{1, 2, ..., n\}$, then the collection $\mathbb{B} = \{B_1 \times B_2 \times ... \times B_n : B_i \in \mathscr{B}_i$ for $i = 1, 2, ..., n\}$ is a base for the product space $X_1 \times X_2 \times ... \times X_n$.