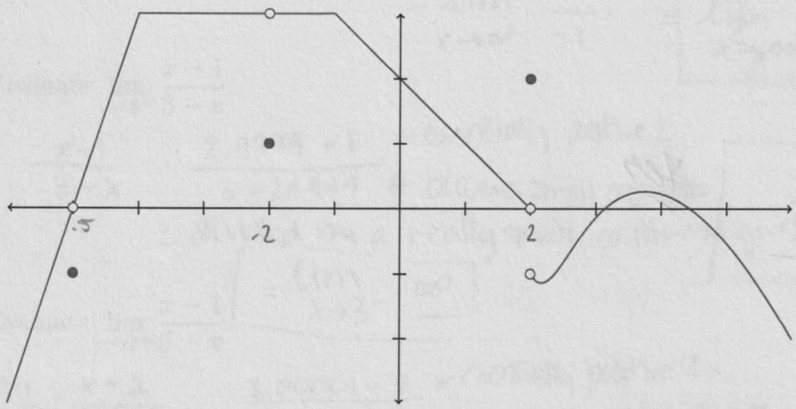
Exam 1 **Calc 1** **9/13/24**

Each problem is worth 10 points. For full credit provide good justification for your answers.

Use the graph of $f(x)$ for problems 1 and 2:



1. (a) What is $\lim_{x \to 2^+} f(x)$? -1

   (b) What is $\lim_{x \to 2^-} f(x)$? 0

   (c) What is $\lim_{x \to 2} f(x)$? DNE because the values as x approaches 2 are different from the left and right

   (d) What is $\lim_{x \to -5^+} f(x)$? 0

   (e) What is $\lim_{x \to -5^-} f(x)$? 0

   Great!

   (f) What is $\lim_{x \to -5} f(x)$? 0 because the values as x approaches -5 are the same from the left and right.

2. Use interval notation to indicate where the function above is continuous.

$(-\infty, -5) \cup (-5, -2) \cup (-2, 2) \cup (2, \infty)$

Good

3. Evaluate $\lim_{x \to 2} \frac{x-2}{x^2-2x}$.

$x^2 - 2x = x^2 - 2x + 0 = (x-2)(x+0)$

$\lim_{x \to 2} \frac{\cancel{x-2}}{\cancel{(x-2)}(x+0)} = \lim_{x \to 2} \frac{1}{x+0} = \frac{1}{2}$

Check for reasonable answer using f(1.9)

$\frac{1.9-2}{1.9^2 - 2(1.9)} = \frac{-0.1}{3.61-3.8} = \frac{-0.1}{-0.19} = 0.5263157$

Excellent!

f(1.9) is close to $\frac{1}{2}$, so $\lim_{x \to 2} \frac{x-2}{x^2-2x} = \frac{1}{2}$ is reasonable.

4. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

x	1	2	3	4	5	6
$f(x)$	9	7	4	5	6	1
$g(x)$	1	8	2	6	5	4

(a) $f(5)$

6

(b) $f(2) - 1$

6

(c) $f(2-1)$

9

Good

(d) $(f \circ g)(3)$

$f(g(3)) = f(2) = \boxed{7}$

(e) $(g \circ f)(3)$

$g(f(3)) = g(4) = \boxed{6}$

5. (a) Evaluate $\lim_{x\to\infty} \frac{x-1}{3-x}$.

$\lim_{x\to\infty} \frac{x-1}{3-x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}} = \lim_{x\to\infty} \frac{1-\frac{1}{x}}{\frac{3}{x}-1} = \frac{1}{-1} = \underline{-1}$

0

Excellent!

(b) Evaluate $\lim_{x\to 3^-} \frac{x-1}{3-x}$.

I know that because the denominator is approaching 0 from positive numbers, $\lim_{x\to 3^-} \frac{x-1}{3-x} = \underline{\infty}$

Input 2.9: $\frac{1.9}{0.1} = 19$ Increases

Input 2.99: $\frac{1.99}{0.01} = 199$ Infinitely

(c) Evaluate $\lim_{x\to 3^+} \frac{x-1}{3-x}$.

I know that because the denominator is approaching 0 from negative numbers, $\lim_{x\to 3^+} \frac{x-1}{3-x} = \underline{-\infty}$

Input 3.1: $\frac{2.1}{-0.1} = -21$ Decreases

input 3.01: $\frac{2.01}{-0.01} = -201$ Infinitely

6. [Stewart 1.3] If a ball is thrown into the air with a velocity of 40 ft/s, its height in feet t seconds later is given by $f(t) = 40t - 16t^2$. Find the average velocity for the time period beginning when $t = 2$ and lasting

$f(2) = 40(2) - 16(2)^2 = 16$

t	$h(t)$
2	16
2.1	13.44
2.5	0

(a) 0.5 second

$f(2.5) = 40(2.5) - 16(2.5)^2 = 0$

$\frac{0-16}{2.5-2} = \frac{-16}{0.5} = \underline{-32 \text{ ft/s}}$

Excellent!

(b) 0.1 second

$f(2.1) = 40(2.1) - 16(2.1)^2 = 13.44$

$\frac{13.44-16}{2.1-2} = \frac{-2.56}{0.1} = \underline{-25.6 \text{ ft/s}}$

7. Biff is a calculus student at Enormous State University, and he's having some trouble. Biff says "Whoa, Calc is kicking my butt! I can plug numbers into functions, you know? But now there's this limit stuff and it's totally confusing because somehow it's not about just plugging one number in. So the professor, he made this big deal about, like, a picture of a cat, right? He said you can't tell how fast the cat is going if you just have a picture of it. I think you can if your camera is good enough, which is why I need to get one of the new iPhones, right? But the professor says definitely you can't tell how fast from just one picture. What's up with that?"

   Help Biff by explaining, in terms he can understand, either how to tell from a good enough picture how fast the cat is moving, or why you're sure it isn't possible.

Well Biff, you'd actually need at least 2 different pictures of the cat. You'd need to know how far apart the cat is in the 2 pictures and how much time went by between the pictures. You see how fast the cat is going is velocity and velocity is how far you go or a distance divided by how long it takes you to get there. It's like a cars speed being measured in miles per hour. It's how far the car is going in miles divided by the number of hours it will take to go that far, In the case of the car it's how far over one hour. The cat will probably only go some number of feet so I'd divide by the number of seconds to make it easier.

Great

8. Evaluate $\lim_{x\to\infty}\left(\sqrt{9x^2+x}-3x\right)$

Great idea!

$$\lim_{x\to\infty}\frac{(\sqrt{9x^2+x}-3x)(\sqrt{9x^2+x}+3x)}{(\sqrt{9x^2+x}+3x)}$$ [Conjugating hehe]

$$\lim_{x\to\infty}\frac{\cancel{9x^2}+x-\cancel{9x^2}}{\sqrt{9x^2+x}+3x}$$

$$\lim_{x\to\infty}\frac{x}{(\sqrt{9x^2+x}+3x)}\frac{1/x}{1/x}$$ [Dividing by 1/x]

$$\lim_{x\to\infty}\frac{x/\cancel{x}}{\frac{\sqrt{9x^2+x}}{x}+\frac{3\cancel{x}}{\cancel{x}}}$$

$$\lim_{x\to\infty}\frac{1}{\frac{\sqrt{9x^2+x}}{\sqrt{x^2}}+3}$$

$$\lim_{x\to\infty}\frac{1}{\sqrt{\frac{9\cancel{x^2}}{\cancel{x^2}}+\frac{x}{x^2}}+3}$$

Excellent!

$$\Rightarrow\frac{1}{\sqrt{9+0}+3}$$

$$\Rightarrow\frac{1}{\sqrt{9}+3}\Rightarrow\frac{1}{3+3}\Rightarrow 1/6\ \#$$

9. Let $f(t) = 40t - 16t^2$. Evaluate $\lim_{x \to 2} \frac{f(x) - f(2)}{x - 2}$.

$$\lim_{x \to 2} \frac{(40x - 16x^2) - (40(2) - 16(2)^2)}{x-2} = \lim_{x \to 2} \frac{40x - 16x^2 - 16}{x-2}$$

$$= \lim_{x \to 2} \frac{-16\cancel{(x-2)}(x-.5)}{\cancel{x-2}} = \lim_{x \to 2} \frac{-16(x-.5)}{1} = \lim_{x \to 2} -16(2) - .5) = \boxed{-24}$$

Excellent!

$-16(x^2 - 2.5x + 1)$

$-16(x-2)(x-.5)$

$-16(x^2 - .5x - 2x + 1)$

10. Evaluate $\lim_{x\to 0} \frac{|x|}{x}$.

$f(-1) = -1$ $\quad f(-0.1) = -1$

$f(1) = 1$ $\quad f(0.1) = 1$

$$\lim_{x\to 0^+} \frac{|x|}{x} = 1$$

Good!

$$\lim_{x\to 0^-} \frac{|x|}{x} = -1$$

$$\lim_{x\to 0} \frac{|x|}{x} = \text{DNE}$$