

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. State the formal definition of the derivative of a function $f(x)$.

2. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

x	1	2	3	4	5	6
$f(x)$	2	4	3	1	6	5
$g(x)$	5	3	2	6	1	4
$f'(x)$	3	7	4	2	6	5
$g'(x)$	4	1	3	8	10	3

(a) If $h(x) = f(x) \cdot g(x)$, what is $h'(3)$ and why?

(b) If $h(x) = \frac{f(x)}{g(x)}$, what is $h'(2)$ and why?

(c) If $h(x) = f(g(x))$, what is $h'(5)$ and why?

(d) If $h(x) = g(f(x))$, what is $h'(5)$ and why?

3. Write an equation for the line tangent to $f(x) = x^3$ at $(2, 8)$.

4. Prove that if f and g are differentiable functions, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

5. Find the derivative of $f(x) = \frac{\cos x}{\sin x}$.

6. State and prove the Quotient Rule for derivatives.

7. Sam is a calculus student at Enormous State University, and they're having some trouble with derivatives. Sam says "Wow, this derivative stuff is confusing. What really gets me is the product and quotient and chain stuff. So like, we're supposed to work in groups in class, which is sad because none of us know how to do it so we're just confused together. But one person said for sine squared of x we should do the chain rule, and this other person said we should write it as sine times sine and use the product rule. How do you tell which way is right?"

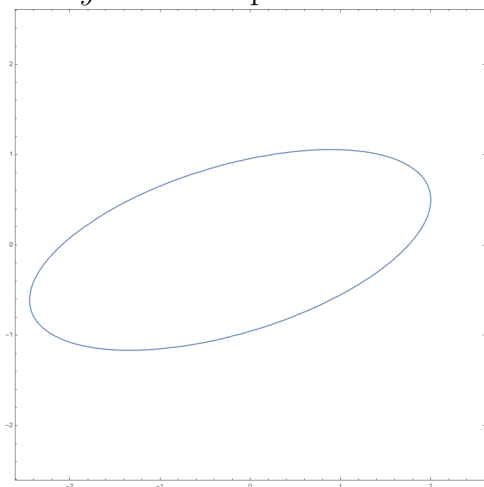
Help Sam by explaining, as clearly as possible, how to decide between their options.

8. (a) Find the linearization $L(x)$ of the function $f(x) = \sqrt{x}$ at $x = 4$.

(b) Use your linearization from part a to approximate $\sqrt{4.1}$.

9. At noon, ship A is 50 nautical miles due west of ship B. Ship A is sailing west at 16 knots and ship B is sailing north at 19 knots. How fast (in knots) is the distance between the ships changing at 3 PM?

10. (a) Find y' for the implicit curve $3x^2 - 6xy + x + 12y^2 = 11$.



- (b) What is the slope of the tangent line to the curve from part a at the point $(2, \frac{1}{2})$?

Extra Credit (5 points possible):

Find *all* points where the tangent line to the curve from #10 is vertical.