Exam 3 **Calc 1** **10/23/24**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. For each for the following functions, give $f'(x)$. No justification required this time.

   (a) $f(x) = \ln x$

   (b) $f(x) = \arcsin x$

   (c) $f(x) = \arctan x$

2. Find an equation for the line tangent to $f(x) = \ln(x)$ at $x = 2$.

3. Use the following table of values for $f(x)$ and $g(x)$ to find values for the following:

x	1	2	3	4	5	6
$f(x)$	2	4	3	1	6	5
$g(x)$	5	3	2	6	1	4
$f'(x)$	3	7	4	2	6	5
$g'(x)$	4	1	3	8	10	3

(a) If $h(x) = \ln(f(x))$, what is $h'(3)$ and why?

(b) If $h(x) = g(x) \cdot \arctan x$, what is $h'(4)$ and why?

4. Why is the derivative of $\ln x$ equal to $\frac{1}{x}$?

5. Evaluate $\lim_{x \to \infty} \frac{3x^2}{e^x}$

6. Why is the derivative of $\arcsin x$ equal to $\dfrac{1}{\sqrt{1-x^2}}$?

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Calculus is hard enough, but there's this really stupid guy in my class who makes it even more impossible! We were supposed to work in pairs on this one problem, and the guy said that $\tan^{-1} x$ could break up into $\dfrac{\sin^{-1} x}{\cos^{-1} x}$ and then we could do this one problem. But I don't even think that's right, is it?"

   Help Bunny understand whether the suggested expression is equivalent.

8. A particular cell phone battery has a peak capacity of 5500mAh at the time of purchase, and after 8 months has a peak capacity of 92% of the original.

   (a) Find a function of the form $f(x) = A\,b^x$ for the peak capacity of the battery after x months have passed.

   (b) What does your formula project the peak capacity will be after 12 months?

   (c) At what rate does your formula predict the peak capacity will be changing after 12 months?

9. If $f(x) = \frac{1}{a} \arctan \frac{x}{a}$, what is $f'(x)$?

10. Can you use the fact that the derivative of a^x is $(\ln a)a^x$ to find the derivative of $\log_a x$?

Extra Credit (5 points possible): Evaluate $\lim_{x \to 1^+} \left(\frac{x}{x-1} - \frac{1}{\ln x} \right)$