Exam 4 **Calc 1** **11/15/24**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Based on the values given in the table,

x	1	2	3	4	5
$f(x)$	-5	-1	-3	1	16
$f'(x)$	9	0	-3	0	9
$f''(x)$	-12	-6	0	6	12

   (a) What are the critical numbers of f?

   (b) Classify each of those critical numbers as a maximum, minimum, or inflection point.

2. Find the interval(s) on which $f(x) = x^3 - 9x^2 + 4$ is decreasing.

3. Find the interval(s) where $f(x) = x^3 - 9x^2 + 4$ is concave up.

4. Find the most general antiderivative of $f(x) = \sqrt[5]{x^2} + x\sqrt{x}$.

5. Find the absolute maximum and absolute minimum values of $f(x) = xe^{-x}$ on $[0, 3]$

6. Two real numbers add up to 25. What is the largest their **product** can be?

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Why do they make it so confusing? I get the slopey parts, you know? But then they have this cavity part, which makes no sense because that's teeth, right? But so somehow the cavity tells you a max instead of a min or something, right? What's up with that?"

   Help Bunny by explaining as clearly as you can how concavity connects to maxes and mins.

8. Approximate $\sqrt[3]{2}$ using Newton's Method with an initial value $x_0 = 2$ to calculate x_1 and x_2.

9. Jon is planning to start selling defective airpods on Temu. His research shows that if he sells them for \$20 he'll sell 500 per month, whereas if he sells them for \$21 he'll sell 450 per month. Assuming that demand is linear, what **price** should he set to maximize his revenue?

10. [WW] A small resort is situated on an island that lies exactly 3 miles from P, the nearest point to the island along a perfectly straight shoreline. 10 miles down the shoreline from P is the closest source of fresh water. If it costs 1.9 times as much money to lay pipe in the water as it does on land, how far down the shoreline from P should the pipe from the island reach land in order to minimize the total construction costs?

Extra Credit (5 points possible): For some values of b the graph of $y = x^4 - bx^2 + 3x + 5$ has two local minimums, and for other values of b it has only one. What can you say about which values of b produce which kind of graph?