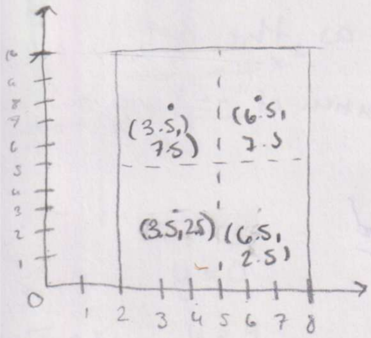
Exam 2 **Calc 3** **10/25/24**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Write a double Riemann sum for $\iint_R f\,dA$, where $R = \{(x,y) : 2 \le x \le 8, 0 \le y \le 10\}$ using midpoints with $n = m = 2$ subdivisions

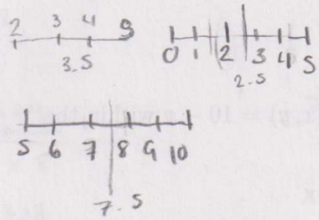


$\Delta x = 3$

$\Delta y = 5$

$\Delta x \, \Delta y = 15$

$= 15\left[f(3.5, 2.5) + f(3.5, 7.5) + f(6.5, 7.5) + f(6.5, 2.5) \right]$



Great

10

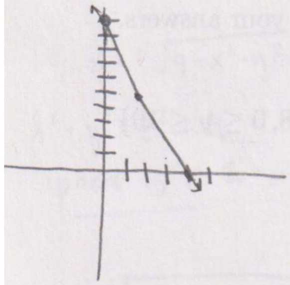
2. Set up a double integral for the integral from #1.

$$\int_2^8 \int_0^{10} f \, dy\,dx$$

10

Good

3. Evaluate the integral $\int_0^4 \int_0^{\frac{-5x}{2}+10} 1\, dy\, dx$

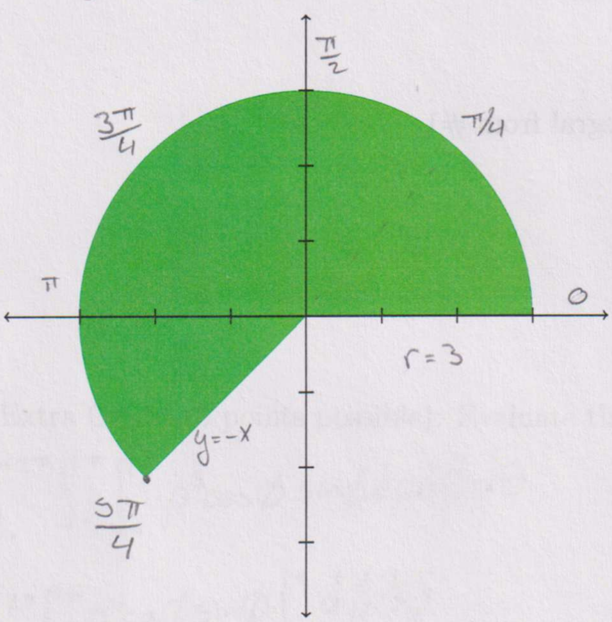


I know that this is a triangle, and I can find the area of a triangle using $\frac{1}{2}bh$, therefore I can apply that here as well.

$\frac{1}{2}(4)(10) = \boxed{20}$

Great!

4. Set up limits of integration for finding the volume under $g(x, y) = 10 - x$ within the toxic green region shown:



$x = r\cos\theta$
$y = r\sin\theta$
$x^2+y^2=r^2$

Polar! Note the jacobian of r

$$\int_0^{\frac{5\pi}{4}} \int_0^3 r(10 - r\cos\theta)\, dr\, d\theta$$

Good

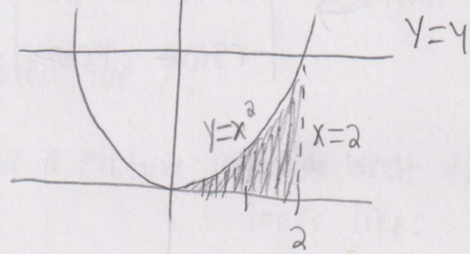
5. Evaluate $\int_0^4 \int_{\sqrt{y}}^2 \sqrt{x^3+1}\,dx\,dy$

Yikes! Let's switch the order of integration

we know $\int_{\sqrt{y}}^{2} dx \implies \sqrt{y} \le x \le 2 \implies y \le x^2 \le 4$

So the limits for y are $0 \le y \le x^2$

Then the x limits should be $0 \le x \le 2$



So the integral becomes

$$\int_0^2 \int_0^{x^2} \sqrt{x^3+1}\,dy\,dx = \int_0^2 \left[y\sqrt{x^3+1} \right]_0^{x^2} dx = \int_0^2 x^2\sqrt{x^3+1}\,dx$$

Let $u = x^3+1$, then $du = 3x^2\,dx$, so $\frac{1}{3}du = x^2 dx$

$$\implies \frac{1}{3}\int_1^9 \sqrt{u}\,du = \frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]\Bigg|_1^9 = \boxed{\frac{1}{3}\left[\frac{2}{3}\left(9^{3/2} - 1^{3/2}\right)\right]}$$

Great

6. Show that the Jacobian for the conversion from rectangular to polar coordinates is what it is.

$$J = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial y}{\partial u} \\ \frac{\partial x}{\partial v} & \frac{\partial y}{\partial v} \end{vmatrix}$$

$\Rightarrow$ For polar conversion: $x = r\cos\theta$

$y = r\sin\theta$

$$J = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial y}{\partial r} \\ \frac{\partial x}{\partial \theta} & \frac{\partial y}{\partial \theta} \end{vmatrix} = \begin{vmatrix} \cos\theta & \sin\theta \\ -r\sin\theta & r\cos\theta \end{vmatrix}$$

$= r \cdot \cos\theta \cdot \cos\theta + r \cdot \sin\theta \cdot \sin\theta$

Excellent!

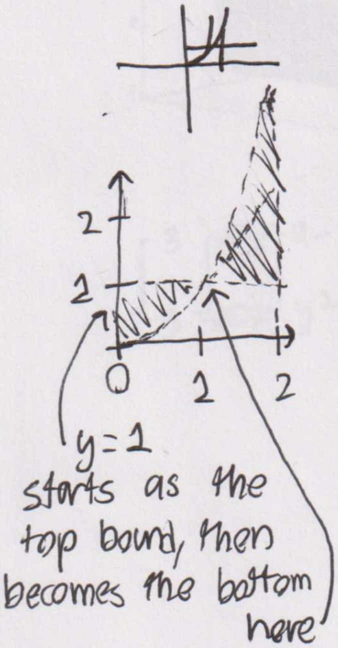
$= r(\cos^2\theta + \sin^2\theta)$

$= r(1) \rightarrow$ by trig identities

$= \underline{\underline{r}}$ ✓

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "OMG! Literally everything is impossible! I set up this one integral, like it was $\int_0^2 \int_1^{x^2} something(x,y)\, dy\, dx$, and the teacher just looked at it for a second and said it was wrong, and he didn't even know which problem I was doing, so evidently it was wrong for everything ever! Then he wouldn't even explain how he could tell it was wrong, he just said I should know! Help!"

Help Bunny by explaining as clearly as you can why the integral she set up needs work.



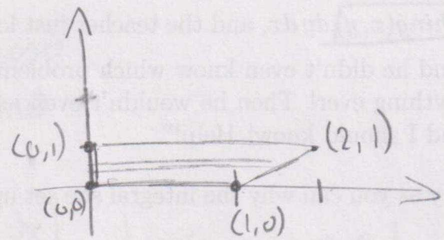
Bunny, this integral is incorrect because your top bound for y actually switches to being the bottom bound halfway! Part of your region is being counted as negative, which will mess up your answer. If you really want to find the value of this region, use two different integrals.

Good.

$\bar{x}$

8. Set up integrals for the x coordinate of the centroid of the trapezoid with vertices $(0,0), (1,0), (2,1)$, and $(0,1)$.

$\boxed{\text{Assume constant density} = k}$



Slope:

$$\frac{1}{2-1} = \frac{1}{1} = 1$$

Point Slope:

$$y - 0 = 1(x-1)$$

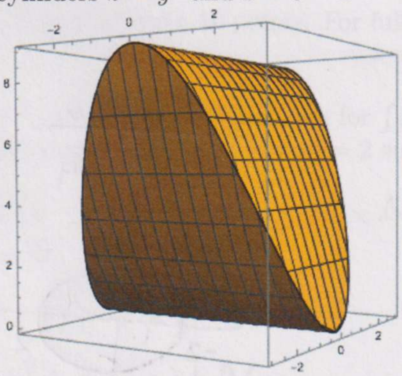
$$y = x - 1$$

$$\Rightarrow x = y + 1$$

Great

$$\bar{x} = \frac{\int_0^1 \int_0^{y+1} k \cdot x \, dx\,dy}{\int_0^1 \int_0^{y+1} k \, dx\,dy}$$

9. Set up an iterated integral for the volume of the region bounded between the parabolic cylinders $z = y^2$ and $z = 9 - x^2$.



$Z = y^2 \qquad Z = 9 - x^2$

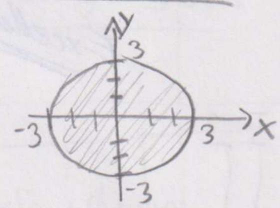
Intersection:

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

$\Rightarrow r = 3$

Top View:



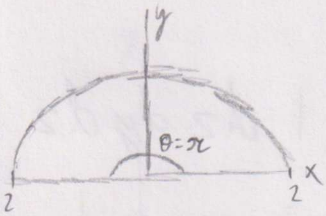
$$\boxed{\int_0^{2\pi}\int_0^{3}\int_{r^2\sin^2\theta}^{9-r^2\cos^2\theta} 1\cdot r \, dz\,dr\,d\theta}$$

Cool.

$Z = r^2\sin^2\theta$

$Z = 9 - r^2\cos^2\theta$

10. Convert the integral $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}}\int_{-\sqrt{4-x^2-y^2}}^{\sqrt{4-x^2-y^2}} z\,dz\,dy\,dx$ to a better coordinate system.



$y=\sqrt{4-x^2}$

Top half of a circle in the xy plane, $y>0$

Yes.

$z=\pm\sqrt{4-x^2-y^2}$

$z^2=4-x^2-y^2$

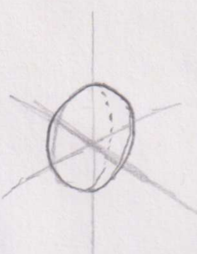
$x^2+y^2+z^2=4$

Sphere with radius 2

$\phi=\pi$

$$\int_0^{\pi}\int_0^{\pi}\int_0^{2} \overset{(z)}{\rho\cos\phi}\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

Excellent!



Front of a sphere