Exam 3 **Calc 3** **11/22/24**

Each problem is worth 10 points. For full credit provide good justification for your answers.

1. Parametrize and give bounds for the bottom half of a sphere with radius 2 centered at the origin.

2. Let $\mathbf{F}$ be the vector field $\mathbf{F}(x, y, z) = 2xy\mathbf{i} + x^2\mathbf{j}$. Let C be the line segment from (2,1) to the origin. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3. Evaluate $\int_C y^3\,dx - x^3\,dy$, where C is the circle $x^2 + y^2 = 4$, oriented counterclockwise.

4. Let $\mathbf{F}(x, y, z) = \langle 2x, 3z, -3z \rangle$, and let S be the cylinder $25 = x^2 + y^2$ between $z = 0$ and $z = 4$, along with a disc of radius 5 in the plane $z = 0$ centered at $(0, 0, 0)$ and another disc of radius 5 in the plane $z = 4$ centered at $(0, 0, 4)$, all with outward orientation. Evaluate $\iint_S \mathbf{F} \cdot \mathbf{dS}$.

5. Let $\mathbf{G}(x, y, z) = \langle 5x, -z, y \rangle$. Let S be the portion of $x^2 + y^2 + z^2 = 4$ above $z = 0$, with upward orientation. Evaluate $\iint_S \text{curl } \mathbf{G} \cdot d\mathbf{S}$.

6. Show that for any scalar function $f(x, y, z)$ with continuous second-order partial derivatives, $\text{curl}(\nabla f) = \mathbf{0}$. Make it clear how the requirement that the partials be continuous is important.

7. Bunny is a calculus student at Enormous State University, and she's having some trouble. Bunny says "Ohmygod. This stuff is so totally confusing. I thought math was where you, like, did problems and stuff, but now there's all this vocabulary stuff, and it's more like French than math or something, you know? So our professor was saying that we should understand why line integrals on closed paths in conservative vector fields are always zero, and I'm not even sure what those words are. Help!"

   Explain clearly to Bunny what the underlined terms mean and why the conclusion is valid.

8. Let $\mathbf{F}(x, y, z) = -y\mathbf{i} + x\mathbf{j}$ and let S be the portion of the cylinder $x^2 + y^2 = 9$ between $z = 0$ and $z = 2$ with outward orientation. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$.

9. Let $\mathbf{F}(x, y) = Q(x, y)\mathbf{j}$. Let C be the line segment from (c, b) to (d, b). Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

10. Let $\mathbf{G}(x, y, z) = \langle 2, z, -y \rangle$ and let S be the portion of the plane $z = 1$ in the first quadrant inside the cylinder $x^2 + y^2 = 9$, with upward orientation. Evaluate $\iint_S \mathbf{G} \cdot d\mathbf{S}$.

Extra Credit (5 points possible): Prove the identity $\operatorname{curl}(f\,\mathbf{F}) = f\,\operatorname{curl}\mathbf{F} + (\nabla f) \times \mathbf{F}$.