Exam 1 **Real Analysis 1** **9/25/24**

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of a sequence $\{a_n\}$ converging.

2. State the definition of an accumulation point.

3. a) State the definition of a Cauchy sequence.

   b) State the Cauchy Convergence Criterion.

4. Give an example of a sequence of positive numbers which is decreasing but not convergent, or say why it's not possible.

5. Prove directly from the definition that $\lim_{x\to\infty} \dfrac{x}{x+1} = 1$.

6. Suppose that $\lim_{x \to a} f(x) = A$ and $\lim_{x \to a} g(x) = B$, where f and g are functions with domain D. Prove (directly from the definition) that $\lim_{x \to a}[f(x) \cdot g(x)] = A \cdot B$.

7. State and prove the Bolzano-Weierstrass Theorem for Sets.

8. Any sequence for which $\lim_{n\to\infty} a_n = L$ for some real limit L must be bounded. Is the same true for a function f if you know $\lim_{x\to\infty} f(x) = L$? Why or why not?

9. Why is the requirement that a be an accumulation point of the domain D included in the definition of the limit of f as x approaches a?

10. Show that for any $x \in \mathbb{R}^+$, there exists $n \in \mathbb{N}$ such that $n - 1 \leq x < n$.