Exam 2 **Real Analysis 1** **11/1/24**

Each problem is worth 10 points. Show adequate justification for full credit. Don't panic.

1. State the definition of the derivative of a function $f(x)$ at $x = a$.

2. State the definition of a function f being continuous at $x = a$.

3. a) State the Intermediate Value Theorem.

b) State Brouwer's Fixed-Point Theorem

4. a) State Fermat's Theorem.

b) State the Mean Value Theorem.

5. a) State the definition of a compact set.

   b) State the Heine-Borel Theorem.

   c) Give an example of an open cover for $\mathbb{R}$ that has no finite subcover.

6. State and prove the Extreme Value Theorem.

7. State and prove Rolle's Theorem.

8. Prove that the product of two functions that are continuous at $x = a$ is continuous at $x = a$.

9. Let E be a set, with $E \subseteq \mathbb{R}$. Show that if E is closed, then $\mathbb{R} \backslash E$ is open.

10. Let $q(x) = \begin{cases} 1 & x \in \mathbb{Q} \\ 0 & x \in \mathbb{R} \backslash \mathbb{Q} \end{cases}$. Where is $x \cdot q(x)$ continuous? Where is it differentiable? Justify your answers well.