Problem Set 2 **Real Analysis 1** **Due 9/9/24**

Five of these problems will be graded, with each problem worth 4 points. Clear and complete justification is required for full credit. LaTeXor equivalent is expected. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Let $A, B \in \mathbb{R}$. Show that $\forall \epsilon > 0, |A - B| < \epsilon \Rightarrow A = B$.
2. Show (carefully) that $|a_n - L| + |a_n - M| < \epsilon/2 + \epsilon/2$ implies $|M - L| < \epsilon$.
3. Suppose that a set S has an infimum. Show that $\inf S$ is unique.
4. Suppose that $\{a_n\}_{n=1}^{\infty}$ is a convergent sequence. Is $\{a_{2n}\}_{n=1}^{\infty}$ convergent as well?
5. Suppose that $\{a_{2n}\}_{n=1}^{\infty}$ is a convergent sequence. Is $\{a_n\}_{n=1}^{\infty}$ convergent as well?
6. The sequence $\{a_n\}$ converges to 0 if and only if the sequence $\{|a_n|\}$ converges to 0.
7. The sequence $\{a_n\}$ converges to A if and only if the sequence $\{|a_n|\}$ converges to $|A|$.
8. Let $\{a_n\}$ be defined by $a_n = \frac{2n}{n+1}$ for $n \in \mathbb{N}$. Show that this sequence converges directly from the definition.
9. If $\{a_n\}$ and $\{b_n\}$ differ from each other in only a finite number of terms, then both sequences converge to the same value or they both diverge. [Kosmala 2.1.8(b)]
10. Give an example of a sequence a_n whose value is 7 for infinitely many values of n, but which does not converge to 7.
11. If $\{a_n\}$ diverges to $+\infty$ and $\{b_n\}$ diverges to $+\infty$, then $\{a_n + b_n\}$ diverges to $+\infty$.
12. If $\{a_n\}$ diverges to $+\infty$ and $\{b_n\}$ diverges to $+\infty$, then $\{a_n - b_n\}$ converges to 0.