Problem Set 4 **Real Analysis 1** **Due 9/20/24**

Five of these problems will be graded, with each problem worth 4 points. LaTeXis expected. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. If a sequence $\{a_n\}$ is bounded and all of its convergent subsequences have the same limit, then $\{a_n\}$ converges.

2. Define a sequence recursively by letting $a_0 = 0$ and letting $a_{n+1} = \sqrt{2 + a_n}\ \forall n \in \mathbb{N}$. Determine whether a_n converges.

3. Suppose that $\lim_{x\to\infty} f(x) = A$, where f is a function with domain D. Prove (directly from the definition) that if k is a constant, $\lim_{x\to\infty}[kf(x)] = kA$.

4. Suppose that $\lim_{x\to\infty} f(x) = A$, where f is a function with domain D. Prove (directly from the definition) that $\lim_{x\to\infty} |f(x)| = |\lim_{x\to\infty} f(x)|$.

5. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, where f and g are functions with domain D. Prove (directly from the definition) that $\lim_{x\to\infty}[f(x) \cdot g(x)] = A \cdot B$.

6. If a limit exists for f as x approaches a, then that limit is unique.

7. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, where f and g are functions with domain D. If $\forall x \in D, f(x) < g(x)$ then $A < B$.

8. Suppose that $\lim_{x\to\infty} f(x) = A$ and $\lim_{x\to\infty} g(x) = B$, where f and g are functions with domain D. If $\forall x \in D, f(x) \leq g(x)$ then $A \leq B$.

9. If $f : D \to \mathbb{R}$ and $\lim f(x)_{x\to a}$ exists, then f is bounded.

10. If $f : D \to \mathbb{R}$ and $\lim f(x)_{x\to a}$ exists, then f is bounded on some set D_1, with $D_1 \subseteq D$.