Problem Set 7 **Real Analysis 1** **Due 10/28/24**

Ten of these problems will be graded, with each problem worth 2 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Let $f(x) = |x|$. Find $f'(0)$, or show it does not exist.

2. Let $f(x) = |x| \cdot x$. Find $f'(0)$, or show it does not exist.

3. Let $f(x) = (x-2)^{2/3}$. Find $f'(2)$, or show it does not exist.

4. Determine where the function

$$f(x) = \begin{cases} x \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable.

5. Determine where the function

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

is differentiable.

6. Prove the Sum Rule for Derivatives. Make it clear why each hypothesis matters.

7. Prove the Quotient Rule for Derivatives. Make it clear why each hypothesis matters.

8. Do Exercise 5.1.6.

9. Do Exercise 5.1.9(a).

10. Prove Corollary 5.2.2(a).

11. Give an example of a function f that is differentiable at $x = a$ such that $f'(a)$ exists, with $f'(a) \neq 0$, but yet f attains a relative extremum at $x = a$.

12. Give an example of a function f that is continuous at $x = a$, not differentiable at $x = a$, but yet f attains a relative extremum at $x = a$.