Problem Set 8 **Real Analysis 1** **Due 11/18/24**

Five of these problems will be graded, with each problem worth 4 points. Clear and complete justification is required for full credit. You are welcome to discuss these problems with anyone and everyone, but must write up your own final submission without reference to any sources other than the textbook and instructor.

1. Let $f(x) = x$, and let $\mathscr{P}$ be a regular partition of $[a, b]$ with n subdivisions. Evaluate $U(\mathscr{P}, f)$ and $L(\mathscr{P}, f)$, and find their limits as n approaches ∞.
2. Suppose that a function $f : [a, b] \to \mathbb{R}$ is bounded and $\mathscr{P} = x_0, x_1, x_2, ..., x_n$ is a partition of $[a, b]$. Then there exist $m, M \in \mathbb{R}$ such that
$$m(b-a) \leq L(\mathscr{P}, f) \leq S(\mathscr{P}, f) \leq U(\mathscr{P}, f) \leq M(b-a)$$
3. Why is the boundedness hypothesis necessary in #2?
4. Suppose that a function $f : [a, b] \to \mathbb{R}$ is bounded and $\mathscr{P}$ and $\mathscr{Q}$ are partitions of $[a, b]$. Show that if $\mathscr{P} \subseteq \mathscr{Q}$, then $U(\mathscr{Q}, f) \leq U(\mathscr{P}, f)$.
5. Why is the boundedness hypothesis required in #4?
6. Show that for any constant $c \in \mathbb{R}$, $f(x) = c$ is Riemann integrable on any interval $[a, b]$ and find the value of $\int_a^b f$.
7. Do Kosmala's Exercise 6.1.4.
8. Do Kosmala's Exercise 6.1.6.
9. Do Kosmala's Exercise 6.2.5. Be clear about how you know your answer is correct.
10. Do Kosmala's Exercise 6.2.11.